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Application of Fuzzy Hamiltonian Graphs for Optimal Route Search in City Transportation

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ABSTRACT

The concept of Fuzzy Hamiltonian graphs in determining optimal routes in city transport network is applied. Traditional graph theory methods have struggled to effectively address imprecise parameters such as fluctuating traffic densities, varying road quality, inconsistent transport costs, and uncertain interconnections between nodes. In the proposed work, the ideas of fuzzy set theory and graph structure are employed to represent transportation network models more accurately. In the suggested framework, cities or junctions are modelled using vertices, while the roads are considered fuzzy edges. The membership values associated with each edge denote accessibility, safety, or transport efficiency levels. The algorithm designed here determines the fuzzy Hamiltonian cycle for finding the best route connecting all important vertices of a graph with minimum uncertainty and transportation cost. The findings show that fuzzy Hamiltonian graphs form a highly versatile mathematical framework for developing efficient route planners. Also, intended for the further expanding the application of FTSP beyond Indian cities to major cities worldwide can enhance route planning.

KEYWORDS: Fuzzy Graphs, Hamiltonian Cycle, Travelling Salesman Problem, Optimal route.

1. INTRODUCTION

Urban transport systems play an important role in economic growth and personal mobility; however, there is usually the problem of traffic jams, unpredictability, complex routes, and variable travel time. Muneera (Muneera et al., 2020) studied the traffic congestion in various Indian cities. Graph theory provides a foundational mathematical framework for modelling real-world transport systems. Networks in transportation can be represented as graphs where nodes represent stops or intersections and edges represent transport modes or connections between locations (Likaj et al., 2013). As the number of vertices and edges increases, it becomes difficult to keep track of all the different ways through which the vertices are connected (Nagoor Gani & Latha, 2016). In real-world applications such as transportation, communication, and vehicle scheduling, edge costs are changeable due to traffic and weather variations, making precise path cost determination extremely difficult (Liao et al., 2021).

Route optimization in urban transportation networks represents a critical challenge in modern logistics and public transportation management. Traditional shortest path problems (SPP) rely on deterministic models that assume precise arc weights and fixed parameters. Fuzzy set theory provides a robust capability for handling uncertain information in transportation contexts (Vel, 2019). Fuzzy logic emerged as a response to fundamental limitations in traditional deterministic and stochastic models when applied to transportation problems (Sarkar Amrita et al., 2012).

Classical route optimization algorithms that are based on graph theory always consider deterministic and exact information in which the distance, cost, or connection between two points is known beforehand (Jain & Kesherwani, 2026). Real-life transport networks, on the other hand, are naturally uncertain owing to factors such as varying traffic, weather conditions, road repairs, incidents, and lack of information. Such uncertainties impose challenges for conventional graph-based algorithms while dealing with actual urban transport problems (Apiecionek, 2025).

The fuzzy graph theory has been an important breakthrough in mathematical studies, being a result of the combination of graph theory and fuzzy set theory (Kumar, 2021). As uncertainty is usually present in many cases regarding graphs, using fuzzy logic is a more suitable choice to address this issue (Sangeetha, 2022).The Graph theory has applications in various fields such as communication, progression in calculations, and data management. Circuit, path, and walk have fantastic applications in resource networking, data storage, and the travelling salesman problem (Murthy et al., 2008). Graphical analysis models have become very popular in modelling highway traffic flow, where their application ranges from regulating traffic flow to optimizing the routes of public transportation. Several researchers have effectively used the Hamilton circuit algorithm in optimizing the route of public transportation (Ratna et al., 2024). (Pandey & Singh, 2023) employs graph theory as its mathematical model to come up with efficient routes, especially for public transport such as bus transport systems used in picking and dropping of students. A possible new route model to be considered in public transport is through the Hamilton Circuit algorithm model where the public transportation route within the city that is single lane (coming and going on the same road) can be replaced by a circular route model where there is distribution of routes evenly across the city (Setiadi, 2018). Dijkstra's algorithm is an integral part of Google Maps when determining the shortest path. In addition, one should note that shortest path problems are central to the use of graph theory, especially with regard to Dijkstra's algorithm (Lavanya et al., 2018). The issue of identifying the dominating nodes in social networks, including Facebook, WhatsApp, and Twitter, an innovative approach to the analysis, is proposed using the influence index and the self-persistence degree of nodes (Ratna et al., 2024). The generalized Travelling Salesman Problem is an extension of the classical Travelling Salesman Problem, where the nodes are partitioned into non-intersecting clusters with the condition that the tour should visit only one node from each cluster (Zorn et al., 2026).

The Fuzzy Technique for Order Preference by Similarity to Ideal Solution (Fuzzy-TOPSIS) is integrated with geographic information systems (GIS) to determine alternative routes by evaluating various criteria such as travel time, speed, distance, environmental impact, and economic feasibility (Madhu et al., 2024). Software tools implementing fuzzy optimization methods enable practical route selection for both public transport authorities and individual drivers. These systems suggest optimal routes accounting for real-world traffic conditions rather than merely shortest geometric paths, providing alternative routes when primary routes experience congestion (Muhamediyeva et al., 2023). The research conducted in Bengaluru also utilizes a combination of the Hamilton circuit approach, along with intelligent agents for optimizing the traffic in urban transit routes and finding out the optimal transit route, which would be better than the currently existing single-lane route through the creation of a circular route (Parkavi & Parthiban, 2025).

While considerable development has taken place in the study areas of fuzzy graph theory, Hamiltonian graph theory, and optimization of transportation networks, previous literature has mostly concentrated either on classical Hamiltonian graphs with deterministic variables or on the concept of fuzzy shortest path routing. The area of research concerning fuzzy Hamiltonian graphs and their applications in urban transportation networks remains considerably underexplored. It is thus important to develop a strong framework based on fuzzy Hamiltonian graphs that can determine optimal paths in urban transportation networks.

2. CONTRIBUTION OF THE PAPER

- The paper makes the contribution to the field of urban transportation by introducing fuzzy logic into graph theory to account for imprecise delays on specific routes and road densities.
- Also, presents efficient algorithms for solving Traveling Salesman Problems and provides improved solutions to transportation problems.
- In addition, the presented algorithms are especially useful because they reduce the computational complexity and allow adapting to the changing traffic conditions to ultimately optimize traffic flow**.**

3. PRELIMINARIES

3.1 Fuzzy Graph: A fuzzy graph is a graph where vertices(nodes) and edges have degree of membership in the range [0,1]. It models systems with uncertainty, imprecision, or partial truth (Bondy & Murty, 1976). Formally, a fuzzy graph $G = (V, \sigma, \mu)$ is defined by:

- $\sigma: V \rightarrow [0,1]$ assigns membership values to vertices.
- $\mu: V \times V \rightarrow [0,1]$ with $\mu(u, v) \leq min(\sigma(u), \sigma(v))$ assigns membership values to edges.

3.2 Path in a Fuzzy Graph: A path defined in a fuzzy graph $\boldsymbol{G = (V, \sigma, \mu)}$ is a sequence of distinct vertices $\boldsymbol{v_1, v_2, v_3, \cdots, v_n (n \geq 2), v_1 \neq v_2}$, such that

$$\boldsymbol{\mu(v_i, v_{i+1}) > 0, i = 1, 2, 3, \cdots, (n-1)} \qquad (1)$$

The two consecutive nodes are the edges of the path. The number of edges, $\boldsymbol{n-1}$, is called the length of the path P (Pal et al., 2020).

3.3 Cycle in a Fuzzy Graph: A path P is a cycle if $\boldsymbol{v_1 = v_2 \ and \ n \geq 3}$. A path $\boldsymbol{P = (V, \sigma, \mu)}$ is a fuzzy cycle if it contains more than one weakest edge.

3.4 Fuzzy Hamiltonian Path: A fuzzy path $P = v_1, v_2, v_3, \cdots, v_n$ where:

- All v_i are distinct

- Each edge $(v_{i,} v_{i+1})$ has non-zero membership $\mu(v_i, v_{i+1}) > 0$.
- The total strength (sum/product of edge memberships) may be considered for optimisation

3.5 Fuzzy Hamiltonian Cycle: A closed fuzzy Hamiltonian path where:

- $v_1 \to v_n$ exists with $\mu(v_n, v_1) > 0$.
- All vertices in V are visited exactly once (no repetition)

3.6 Fuzzy Hamiltonian Graph: Let, $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{\sigma}, \boldsymbol{\mu})$ be a fuzzy graph with a finite set of veritices V. A fuzzy graph that contains at least one fuzzy Hamiltonian cycle (Murali & Rao, 2018).

3.7 Strength of a Path: The strength of a path, denoted by $s(P)$, is defined as the minimum edge membership value along the path:

$$s(P) = \bigwedge_{i=1}^{n} \mu(x_{i-1}, x_i) \tag{2}$$

The strength of connectedness between any two vertices u and v, denoted by $CON_G(u, v)$ is the maximum strength among all possible paths linking them. A fuzzy graph is connected if $CON_G(u, v) > 0$ for each pair of vertices in the active support of σ, denoted as

$$supp(\sigma) = \{v\epsilon V \mid \sigma(v) > 0\} \tag{3}$$

An edge uv is classified as a "strong edge" when its membership value is at least equal to the strength of connectedness between u and v in the auxiliary subgraph resulting from its removal:

$$\mu(uv) \geq CON_{G-uv}(u, v) \tag{4}$$

4. TRAVELLING SALESMAN PROBLEM (TSP)

The Travelling Salesman Problem (TSP) is an optimization problem that is applied in logistics in order to find the shortest possible route for deliveries and pickups to be made to various points and return to the start. The TSP aims at finding the shortest route connecting a number of cities and returning to the initial city (Mezzina & Pavone, 2026). Such algorithms are efficient in balancing between global exploration and local exploitation due to stochastic search procedures, thus making it possible for them to traverse difficult spaces (Almufti & Shaban, 2025).

The application range of TSP includes logistics, circuit routing, scheduling, and, more recently, drones, making TSP a common benchmark tool for testing and improving meta-heuristics. The proposed method involves the enumeration of all initial routes and then optimizing the rest of the route (Makarova et al., 2025).

A proposed algorithm for the travelling salesman problem to find an efficient and cost-effective Hamiltonian tour within the city is given in **algorithm 1**, where the connection between the places is described by the cost matrix.

Algorithm 1: Proposed Algorithm for Finding an Optimal Route Using the Travelling Salesman Problem

Input:

- A cost matrix D of size $n \times n$, where $D[i][j] = 0$ indicates no direct connection between cities i and j.
- A set of cities $C = \{c_1, c_2, \dots, c_n\}$.
- A starting city s.

Output:

- A Hamiltonian tour (if possible).
- Total travel cost of the tour.

```
Algorithm Optimal_Route_Nearest_Neighbour(D, Cities, Start)

1. n ← number of cities
2. visited[1..n] ← FALSE
3. tour ← [Start]
4. visited[Start] ← TRUE
5. current ← Start
6. total_cost ← 0

7. while length(tour) < n do
8.     min_cost ← ∞
9.     next_city ← NULL


10.    for each city i do
11.        if visited[i] = FALSE and
            D[current][i] > 0 and
            D[current][i] < min_cost then
12.            min_cost ← D[current][i]
13.            next_city ← i
14.        end if
15.    end for

16.    if next_city = NULL then
17.         print "No Hamiltonian Tour Possible"
18.         stop
19.    end if

20.    append next_city to tour
21.    visited[next_city] ← TRUE
22.    total_cost ← total_cost + min_cost
23.    current ← next_city

24. end while

25. if D[current][Start] > 0 then
26.     append Start to tour
27.     total_cost ← total_cost + D[current][Start]
```

```
28. end if

29. print tour
30. print total_cost

End Algorithm
```

5. THEORETICAL RESULTS

Some theorems for the existence of a Hamiltonian Cycle in Fuzzy graphs are stated here (Bondy & Murty, 1976):

Theorem 1. Dirac [1952]: If $G = (V, E)$ is a simple graph with $|V| \geq 3$ and $\delta \geq \frac{v}{2}$, then G is Hamiltonian.

Theorem 2. Ore [1962]: Let $G = (V, E)$ be a simple graph and let a and b be non-adjacent vertices in G such that $d(a) + d(b) \geq |V|$. Then G is Hamiltonian if and only if $G + ab$ is Hamiltonian.

Theorem 3. Nash-Williams [1969]: Let $G = (V, E)$ be a k-regular graph on 2k+1 nodes. Then G is a Hamiltonian graph.

Theorem 4. Erdos and Hobbs [1978]: Let $G = (V, E)$ be a 2-connected, k-regular graph on 2k+4 nodes, where $k \geq 4$. Then G is a Hamiltonian graph.

Theorem 5. Bollobas and Hobbs [1978]: Let G be a 2-connected, k-regular graph on n vertices, where $9k/4 \geq n$. Then G is a Hamiltonian graph.

6. RESEARCH METHODOLOGY

The Hamilton circuit is a path that passes through each node in the graph exactly once, except the origin, which is passed twice (Murali & Rao, 2018).

The number of possible Hamilton circuits *N*, can be calculated using the formula (Vijaya & Kannan, 2017):

$$N = \frac{1}{2}(n-1)! \tag{5}$$

where n represents the number of vertices(nodes) on the graph.

The methodology adopted in this study combines concepts from Fuzzy Set Theory (Zadeh et al., 1965) and the concept of fuzziness (Kharsyntiew, 2025). Fuzzy set theory has demonstrated promise for solving multimodal transport network problems with uncertain data, particularly when edge costs are represented as fuzzy numbers (Verga et al., 2013). Graph Theory and Optimization Techniques for modelling and analysing the connectivity in the city of Indore. The research work aims at developing a Connected Fuzzy Hamiltonian Graph by using geographical locations and transportation links obtained from the map of the city. There were various steps involved in the approach, such as graph formation, fuzzy edge identification, analysis of graph connectivity, Hamiltonian Path formation, and solving the Travelling Salesman Problem.

7. CONSTRUCTION OF THE FUZZY GRAPH

A fuzzy graph was constructed by considering transportation routes between locations as edges. Since transportation connectivity in real-world road networks is uncertain and dynamic due to traffic density, road conditions, and route accessibility, fuzzy membership values were assigned to each edge.

The fuzzy graph is represented as:

$$G_f = (V, \sigma, \mu) \tag{6}$$

where:

- V is the set of vertices,
- $\sigma(v)$ represents the membership value of vertices,
- $\mu(u, v)$ represents the fuzzy membership value of the edge between vertices u and v.

Taking,

$$\sigma(v) = 1, \forall\, v \in V \tag{7}$$

indicating that all vertices fully exist in the network.

Fuzzy edge membership values were assigned in the interval:

$$0 \leq \mu(u, v) \leq 1$$

based on the approximate transportation strength observed from the map. Higher membership values indicate stronger and more direct connectivity between locations.

In transit routing, routing engines prioritize effective paths constructed entirely of strong edges to ensure that the route does not suffer from unexpected severe structural capacity drops.

Problem Statement

A map of Indore city is considered here for the underlying transportation system. The critical places observed on the map were considered as nodes of the graph. The selected places included:

$$V = \{a, i, l, c, b, h, k, m\} \tag{8}$$

where:

a= Airport, i= Indore City Center, l= Lal Bagh, c= Choithram, b= Bilawali, h= Khajrana Temple, k= Kanadiya, m= Muskhedi.

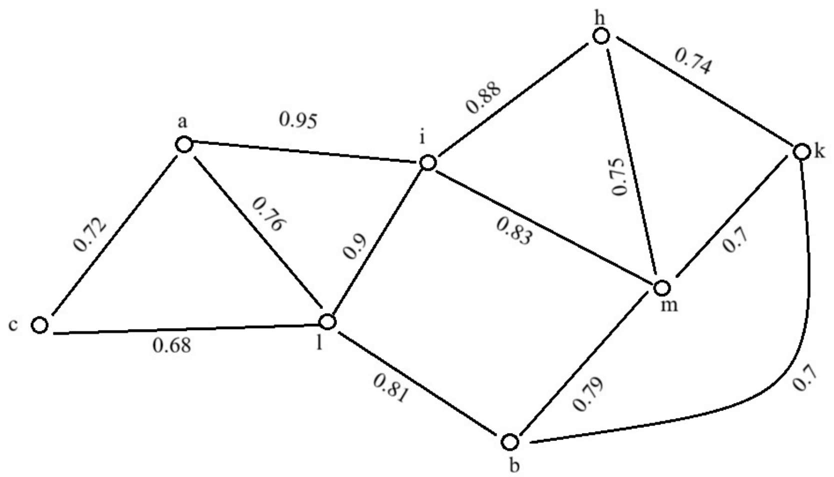

fig 5.1 : A connected graph depicting various places of Indore City

In fig 5.1, various places of the Indore city are taken as nodes, and the edges connecting them are considered the routes between the places having fuzzy membership values as weights.

Define,

$$cost(a,b) = \frac{1}{\mu(a,b)} \tag{9}$$

Consider Table 2 for the relation between the edge membership and the cost of traversing each edge.

Table 1: Relationship between edge membership and cost between edges

$\boldsymbol{Edge}$	a–i	a–c	a–l	i–l	i–h	i–m	l–b	b–m	m–h	h–k	b–k	c–l	m-k
$\boldsymbol{\mu}$	0.95	0.72	0.76	0.90	0.88	0.83	0.81	0.79	0.75	0.74	0.70	0.68	0.70
$\boldsymbol{cost = 1/\mu}$	1.05	1.39	1.32	1.11	1.14	1.20	1.23	1.27	1.33	1.35	1.43	1.47	1.43

We are implementing the **algorithm 1**, by utilizing a Python script that accepts the cost matrix as input to identify the optimal Hamiltonian Path in Indore. The script is presented in Table 2.

Table 2: A Python code to find the Hamiltonian path in the given city

```
# Cost matrix (zeros represent NO connection)
cost = [
    [0, 1.05, 1.32, 1.39, 0, 0, 0, 0],
    [1.05, 0, 1.11, 0, 0, 1.14, 0, 1.20],
    [1.32, 1.11, 0, 1.47, 1.23, 0, 0, 0],
    [1.39, 0, 1.47, 0, 0, 0, 0, 0],
    [0, 0, 1.23, 0, 0, 0, 1.43, 0, 1.27],
    [0, 1.14, 0, 0, 0, 0, 1.35, 1.33],
    [0, 0, 0, 0, 1.43, 1.35, 0, 1.43],
    [0, 1.20, 0, 0, 1.27, 1.33, 1.43, 0],
]

cities = ["a", "i", "l", "c", "b", "h", "k", "m"]

# Select origin city
start = 3

n = len(cities)
visited = [False] * n
tour = [start]
visited[start] = True

current = start
total_cost = 0

print("Origin City:", cities[start])
```

```
# Visit all cities - KEY CHANGE: skip zero cost
while len(tour) < n:
    min_dist = float('inf')
    next_city = None

    for i in range(n):
        # CRITICAL: Check cost > 0 (no connection if zero)
        if not visited[i] and cost[current][i] > 0 and cost[current][i] < min_dist:
            min_dist = cost[current][i]
            next_city = i

    if next_city is None:
        print(f"\nERROR: No valid connection from {cities[current]} to unvisited cities!")
        print("Graph is not fully connected. Cannot complete Hamiltonian tour.")
        break

    tour.append(next_city)
    visited[next_city] = True
    total_cost += min_dist
    current = next_city

# Return to origin (also check > 0)
if len(tour) == n:
    return_dist = cost[current][start]
    if return_dist > 0:
        total_cost += return_dist
        tour.append(start)

# Display result
print("\nHamiltonian Tour:")
print(" -> ".join(cities[i] for i in tour))
print("\nTotal Cost =", total_cost)
```

Output:

Hamiltonian Tour:

$c \to a \to i \to l \to b \to k \to h \to m$

Total Cost = 8.889999999999999

Procedure

Step1. An adjacency matrix of the given figure of the corresponding city is constructed.

Step2. Select an origin place(node) to start the tour.

Step3. For the next node, check either the $cost > 0$ and the minimum among the other nodes connected.

Step4. Repeat step 3 until all nodes are covered, and we get a Hamiltonian path.

Step5. To find the total cost of traversal, add the weight assigned to each edge.

The obtained Hamiltonian Cycle is:

$$c \to a \to i \to l \to b \to k \to h \to m$$

Total Cost

$$1.39 + 1.05 + 1.11 + 1.23 + 1.43 + 1.35 + 1.33 = 8.89$$
$$cost = 8.89 \tag{10}$$

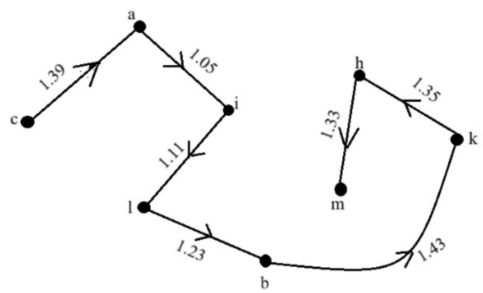

fig 5.2

Hamiltonian route traversed

8. DISCUSSION

Hamiltonian paths were obtained using the given algorithm for the city map of Indore; however, since the given algorithm relies on the selection of the initial node, it is not always possible to form a Hamiltonian path for all the nodes, which necessitates further study. When the problem is modelled as the Fuzzy Travelling Salesman Problem (FTSP), solutions are possible because the problem considers the environment's vagueness in terms of uncertainty or fuzziness. Applications of FTSP can be extended from Indian cities to other metropolises around the world.

Using hybrid models that integrate fuzzy logic's ability to represent uncertainty and neural networks' ability to learn from data can benefit FTSP algorithms in a variety of ways. The combination is capable of learning complicated nonlinear dependencies from past data as well as choosing the nearly optimal path based on current conditions, using interpretable fuzzy memberships for this purpose. The resulting approach makes the solution more robust to noisy input or incomplete data (e.g., travel times, road closures, faulty sensors, and so forth).

However, there are still some issues that need to be addressed. The output Hamiltonian tour obtained from the algorithm may differ based on the initial node used, thus impacting its optimality and consistency; hence, modifications must be made to the algorithm, such as using the multiple-start approach or making use of normalized start nodes and metaheuristic techniques. The complexity of computing also increases as TSP and FTSP variations fall under the category of NP-hard problems, and as such, problem expansion would require heuristic algorithms or problem decomposition. Another issue that must be taken into consideration in using fuzzy modelling is the correct specification of membership functions and fuzzy rules.

9. CONCLUSION

Based on the results and methods carried out, the Travelling Salesman Problem (TSP) serves as an effective model for determining an optimal route that minimizes the total travel cost. This approach ensures that the selected path covers all designated locations with the least possible expenditure in terms of distance, time, or other relevant cost metrics. However, it is important to note that while alternative routes may exist that offer similarly low costs, these alternatives might not satisfy the Hamiltonian property, meaning they do not visit each location exactly once without repetition.

In this study, a subset of key locations within Indore city has been selected to demonstrate the application of TSP in a practical urban context. The methodology can be scaled by incorporating all major points of interest or essential destinations across the city, thereby providing a comprehensive route that visits each place exactly once while minimizing the overall travel cost.

10. CONFLICT OF INTEREST

The authors declare that there are no financial, commercial, institutional, or personal relationships that could have influenced the work reported in this research.

11. RESEARCH INTEGRITY STATEMENT

The authors declare that the manuscript is original, complies with the journal's plagiarism and AI usage requirements, has not been published or submitted elsewhere, and that all applicable ethical guidelines have been followed in conducting, reporting, and publishing the research.

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